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# Rayleigh–Taylor instability in a viscoelastic binary fluid

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The effects of polymer additives on Rayleigh–Taylor (RT) instability of immiscible fluids is investigated using the Oldroyd-B viscoelastic model. Analytic results obtained exploiting the phase-field approach show that in polymer solution the growth rate of the instability speeds up with elasticity (but remains slower than in the pure solvent case). Numerical simulations of the viscoelastic binary fluid model confirm this picture.

**Key words:** Rayleigh–Taylor instability, viscoelastic fluids

## 1. Introduction

Mixing of species (e.g. contaminants, tracers and particles) and thermodynamical quantities (e.g. temperature) are dramatically influenced by fluid flows (Dimotakis 2005). Controlling the rate of mixing in a flow is an objective of paramount importance in many fields of science and technologies with wide-ranging consequences in industrial applications (Warnatz, Maas & Dibble 2001).

The difficulties of the problem come from the intricate nature of the underlying fluid flow, which involves many active nonlinearly coupled degrees of freedom (Frisch 1995), and on the poor comprehension of the way through which the fluid is coupled to the transported quantities. The problem is even more difficult when the transported quantity reacts back to the flow field thus affecting its dynamics. An instance is provided by the heat transport in convection (Siggia 1994).

Mixing emerges as a final stage of successive hydrodynamic instabilities (Drazin & Reid 1981) eventually leading to a fully developed turbulent stage. The possibility of controlling such instability mechanisms thus allows one to have a direct control on the mixing process. In some cases the challenge is to enhance the mixing process by stimulating the turbulence transition, in yet other cases the goal is to suppress deleterious instabilities and the ensuing turbulence. Inertial confinement fusion (Cook & Zhou 2002) is an example whose success relies on the control of the famous Rayleigh–Taylor (RT) instability occurring when a heavy, denser, fluid is accelerated into a lighter one. For a fluid in a gravitational field, such instability was first described Lord Rayleigh in the 1880s (Rayleigh 1883) and later generalized to all accelerated fluids by Sir Geoffrey Taylor in 1950 (Taylor 1950).

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Our attention here is focused on RT instability with the aim of enhancing the perturbation growth rate in its early stage of evolution. The idea is to inject polymers into the fluid and to study on both analytical and numerical ground how the stability of the resulting viscoelastic fluid is modified. Similar problems were already investigated in more specific context, including RT instability of viscoelastic fluids with suspended particles in porous medium with a magnetic field (Sharma & Rajput 1992) and RT linear stability analysis of viscoelastic drops in high-speed airstream (Joseph, Beavers & Funada 2002). We also mention that the viscoelasticity is known to affect also other kinds of instabilities, including Saffman–Taylor instability (Wilson 1990; Coussot 1999), Faraday waves (Muller & Zimmermann 1999; Wagner, Muller & Knorr 1999), the stability of Kolmogorov flow (Boffetta *et al.* 2005), Taylor–Couette flow (Larson, Shaqfeh & Muller 1990; Groisman & Steinberg 1996) and Rayleigh–Bénard problem (Vest & Arpaci 1969; Sokolov & Tanner 1972).

The paper is organized as follows. In § 2 the basic equations ruling the viscoelastic immiscible RT system are introduced together with the phase-field approach. In § 3 the linear analysis is presented and the analytical results shown and discussed in § 4. The resulting scenario is corroborated in § 5 by means of direct numerical simulations of the original field equations.

## 2. Governing equations

The system we consider is composed of two incompressible fluids (labelled by 1 and 2) having different densities, $\rho_1$ and $\rho_2 > \rho_1$, and different dynamical viscosities, $\mu_1$ and $\mu_2$, with the denser fluid placed above the less dense one. For more generality, the two fluids are supposed to be immiscible so that the surface tension on the interface separating the two fluids will be explicitly taken into account.

The effects of polymer additives is here studied within the framework of the Oldroyd-B model (Oldroyd 1950; Hinch 1977; Bird *et al.* 1987). In this model polymers are treated as elastic dumbbells, i.e. identical pairs of microscopic beads connected by harmonic springs. Their concentration is supposed to be low enough to neglect polymer–polymer interactions. The polymer solution is then regarded as a continuous medium, in which the reaction of polymers on the flow is described as an elastic contribution to the total stress tensor of the fluid (Bird *et al.* 1987).

In order to describe the mixing process of the resulting viscoelastic immiscible fluids we follow the phase-field approach (for a general description of the method see, e.g. Cahn & Hilliard 1958; Bray 2002, and for application to multiphase flows see, e.g. Badalassi, Ceniceros & Banerjee 2003; Ding, Spelt & Shu 2007; Morro 2007; Celani *et al.* 2009). Here, we only recall that the basic idea of the method is to treat the interface between two immiscible fluids as a thin mixing layer across which physical properties vary steeply but continuously. The evolution of the mixing layer is ruled by an order parameter (the phase field) that obeys a Cahn–Hilliard equation (Cahn & Hilliard 1958). One of the advantage of the method is that the boundary conditions at the fluids interface need not to be specified being encoded in the governing equations. From a numerical point of view, the method permits to avoid a direct tracking of the interface and easily produces the correct interfacial tension from the mixing-layer free energy.

To be more specific, the evolution of the viscoelastic binary fluid is described by the system of differential equations:

$$\rho_0\left(\partial_t \boldsymbol{v} + \boldsymbol{v}\cdot\boldsymbol{\partial}\boldsymbol{v}\right) = -\boldsymbol{\partial} p + \boldsymbol{\partial}\cdot(2\mu\boldsymbol{e}) + A\rho_0\boldsymbol{g}\phi - \phi\boldsymbol{\partial}\mathscr{M} + \frac{2\mu\eta}{\tau}\boldsymbol{\partial}\cdot(\boldsymbol{\sigma} - \mathbb{I}), \qquad (2.1)$$

$$\partial_t \phi + \boldsymbol{v} \cdot \boldsymbol{\partial} \phi = \gamma \partial^2 \mathscr{M}, \tag{2.2}$$

$$\partial_t \boldsymbol{\sigma} + \boldsymbol{v} \cdot \boldsymbol{\partial} \boldsymbol{\sigma} = (\boldsymbol{\partial} \boldsymbol{v})^T \cdot \boldsymbol{\sigma} + \boldsymbol{\sigma} \cdot \boldsymbol{\partial} \boldsymbol{v} - \frac{2}{\tau}(\boldsymbol{\sigma} - \mathbb{I}). \tag{2.3}$$

Equation (2.1) is the usual Boussinesq Navier–Stokes equation (Kundu & Cohen 2001) with two additional stress contributions. The first one arises at the interface where the effect of surface tension enters into play (Bray 2002; Yue *et al.* 2004; Berti *et al.* 2005), the last term represents the polymer back reaction to the flow field (Bird *et al.* 1987).

In (2.1), we have defined $\rho_0 = (\rho_1 + \rho_2)/2$, $\boldsymbol{g}$ is the gravitational acceleration pointing along the $y$-axis, $A \equiv (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$ is the Atwood number, $e_{ij} \equiv (\partial_i v_j + \partial_j v_i)/2$ is the rate of strain tensor and $\mu = \mu(\phi)$ is the dynamical viscosity field parametrically defined as (Liu & Shen 2003):

$$\frac{1}{\mu} = \frac{1+\phi}{2\mu_1} + \frac{1-\phi}{2\mu_2}, \tag{2.4}$$

$\phi$ being the phase field governed by (2.2). The phase field $\phi$ is representative of density fluctuations and we take $\phi = 1$ in the regions of density $\rho_1$ and $\phi = -1$ in those of density $\rho_2 \geqslant \rho_1$. $\boldsymbol{\sigma} \equiv \langle \boldsymbol{R}\boldsymbol{R} \rangle / R_0^2$ is the polymer conformation tensor, $\boldsymbol{R}$ being the end-to-end polymer vector ($R_0$ is the polymer length at equilibrium), the parameter $\eta$ is proportional to polymer concentration and $\tau = \tau(\phi)$ is the (slowest) polymer relaxation time which, according to the Zimm model (Doi & Edwards 1986), is assumed to be proportional to the viscosity $\mu$ (therefore we have $\tau = \tau_1$ for $\phi = 1$ and $\tau = \tau_2$ for $\phi = -1$ with $\mu(\phi)/\tau(\phi)$ constant). Finally, $\gamma$ is the mobility and $\mathscr{M}$ is the chemical potential defined in terms of the Ginzburg–Landau free energy $\mathscr{F}$ as (Cahn & Hilliard 1958; Bray 2002; Yue *et al.* 2004):

$$\mathscr{M} \equiv \frac{\delta \mathscr{F}}{\delta \phi} \quad \text{and} \quad \mathscr{F}[\phi] \equiv \lambda \int_{\Omega} \mathrm{d}\boldsymbol{x} \left( \frac{1}{2} |\boldsymbol{\partial} \phi|^2 + V(\phi) \right), \tag{2.5}$$

where $\Omega$ is the region of space occupied by the system, $\lambda$ is the magnitude of the free-energy and the potential $V(\phi)$ is

$$V(\phi) \equiv \frac{1}{4\epsilon^2}(\phi^2 - 1)^2, \tag{2.6}$$

where $\epsilon$ is the capillary width, representative of the interface thickness.

The unstable equilibrium state with heavy fluid placed on the top of light fluid is given by

$$\boldsymbol{v} = 0, \quad \phi(y) = -\tanh\left(\frac{y}{\epsilon\sqrt{2}}\right) \quad \text{and} \quad \boldsymbol{\sigma} = \mathbb{I} \tag{2.7}$$

corresponding to a planar interface of width $\epsilon$ with polymers having their equilibrium length $R_0$. In this case, the surface tension $\mathscr{S}$ is given by (see e.g. Landau & Lifshitz 2000):

$$\mathscr{S} \equiv \lambda \int_{-\infty}^{+\infty} \mathrm{d}y \left( \frac{1}{2} |\boldsymbol{\partial} \phi|^2 + V(\phi) \right) = \frac{2\lambda\sqrt{2}}{3\epsilon}. \tag{2.8}$$

The sharp-interface limit is obtained by taking the $\lambda$ and $\epsilon$ to zero, keeping $\mathscr{S}$ fixed to the value prescribed by surface tension (Liu & Shen 2003).

## 3. Linear stability analysis

Let us now suppose to impose a small perturbation on the interface separating the two fluids. Such perturbation will displace the phase field from the previous equilibrium configuration, which minimizes the free energy (2.5) to a new configuration for which, in general, $\mathscr{M} \neq 0$. We want to determine how the perturbation evolves in time.

Focusing on the two-dimensional case (corresponding to translational invariant perturbations along the $z$ direction), let us denote by $h(x,t)$ the perturbation imposed to the planar interface $y=0$ in a way that we can rewrite the phase-field $\phi$ as

$$\phi = f\left(\frac{y-h(x,t)}{\epsilon\sqrt{2}}\right), \tag{3.1}$$

where $h$ can be larger than $\epsilon$, yet it has to be smaller than the scale of variation of $h$ (small amplitudes). In this limit we assume the interface to be locally in equilibrium, i.e. $\partial^2 f/\partial y^2 = V'(f)$, and thus $f(y) = -\tanh(y)$ and therefore $\mathscr{M} = -\lambda(\partial^2 f/\partial x^2)$ ($'$ denotes derivative with respect to the argument).

Linearizing the momentum equation for small interface velocity, we have

$$\rho_0 \partial_t v_y = -\partial_y p - \phi\partial_y \mathscr{M} - Ag\rho_0\phi + \frac{2\mu\eta}{\tau}\partial_i\sigma_{i2} + \mu\left(\partial_x^2 + \partial_y^2\right)v_y + 2(\partial_y v_y)\partial_y\mu. \tag{3.2}$$

Integrating on the vertical direction and using derivations by parts one gets

$$\rho_0\partial_t q = \mathscr{S}\frac{\partial^2 h}{\partial x^2} + 2Ag\rho_0 h + \frac{2\mu\eta}{\tau}\Sigma + Q, \tag{3.3}$$

where we have defined

$$Q \equiv \int_{-\infty}^{+\infty} \mu\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right) v_y \,\mathrm{d}y, \quad q \equiv \int_{-\infty}^{\infty} v_y \,\mathrm{d}y, \quad \Sigma \equiv \int_{-\infty}^{\infty} \partial_x \sigma_{12}\,\mathrm{d}y, \tag{3.4}$$

and we have used the relations $\int (f')^2\,\mathrm{d}y = 2\sqrt{2}/(3\epsilon)$, $\int ff'''\,\mathrm{d}y = 0$, $\int f\,\mathrm{d}y = 2h$.

Note that, unlike what happens in the inviscid case, (3.3) does not involve solely the field $q_y$ but also second-order derivatives of $v_y$. In order to close the equation, let us resort to a potential-flow description. The idea is to evaluate $Q$ for a potential flow $v_y$ and then to plug $Q = Q^{pot}$ into (3.3) (Mikaelian 1993). The approximation is justified when viscosity is sufficiently small and its effects are confined in a narrow region around the interface. Because for a potential flow $\partial^2 \boldsymbol{v} = 0$, we have

$$Q^{pot} = 2\int_{-\infty}^{+\infty}\mu\frac{\partial^2 u_y}{\partial x^2}\,\mathrm{d}y = 2\int_{-\infty}^{0}\mu\frac{\partial^2 u_y}{\partial x^2}\,\mathrm{d}y + 2\int_{0}^{\infty}\mu\frac{\partial^2 u_y}{\partial x^2}\,\mathrm{d}y = (\mu_1+\mu_2)\frac{\partial^2 q}{\partial x^2}. \tag{3.5}$$

Substituting in (3.3) and defining $\nu = (\mu_1+\mu_2)/(2\rho_0)$, one finally obtains

$$\partial_t q = \frac{\mathscr{S}}{\rho_0}\frac{\partial^2 h}{\partial x^2} + 2Agh + \frac{2\mu\eta}{\tau\rho_0}\Sigma + 2\nu\frac{\partial^2 q}{\partial x^2}. \tag{3.6}$$

Let us now exploit (2.2) for the phase field to relate $q_y$ to $h$. For small amplitudes, we have

$$\partial^2\mathscr{M} = \frac{\lambda}{\epsilon\sqrt{2}}\left[f'\frac{\partial^4 h}{\partial x^4} + \frac{1}{2\epsilon^2}f'''\frac{\partial^2 h}{\partial x^2}\right] \tag{3.7}$$

and therefore, from (2.2),

$$-\frac{1}{\epsilon}f'\partial_t h + v_y\frac{1}{\epsilon}f' = \frac{\gamma\lambda}{\epsilon}\left[f'\partial_x^4 h + \frac{1}{2\epsilon^2}f'''\partial_x^2 h\right]. \tag{3.8}$$

Integrating over $y$, observing that $1/(2\sqrt{2}\epsilon)f'$ approaches $\delta(y-h)$ as $\epsilon \to 0$ and using the limit of sharp interface ($\gamma\lambda \to 0$), one obtains

$$\partial_t h = v_y(x, h(t,x), t) \equiv v_y^{(int)}(x,t). \tag{3.9}$$

The equation for the perturbation $\sigma_{12}$ of the conformation tensor is obtained by linearizing (2.3) around $\sigma_{\alpha\beta} = \delta_{\alpha\beta}$:

$$\partial_t \sigma_{12} = \partial_x v_y + \partial_y v_x - \frac{2}{\tau}\sigma_{12} \tag{3.10}$$

from which, exploiting incompressibility, we obtain

$$\partial_t \partial_x \sigma_{12} = \left(\partial_x^2 - \partial_y^2\right) v_y - \frac{2}{\tau}\partial_x \sigma_{12} - 2\sigma_{12}\partial_x \frac{1}{\tau}. \tag{3.11}$$

For small-amplitude perturbations the last term, which is proportional to $\sigma_{12}\partial_x\phi$, can be neglected at the leading order. Integrating over $y$ and using again the potential flow approximation one ends up with

$$\partial_t \Sigma = 2\partial_x^2 q - \frac{2}{\bar{\tau}}\Sigma - \left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right) \int \mathrm{d}y \phi \partial_x \sigma_{12}, \tag{3.12}$$

where we have introduced $\bar{\tau} = 2\tau_1\tau_2/(\tau_1 + \tau_2)$.

In conclusion, we have the following set of equations (in the $(x, t)$ variables) for the linear evolution of the RT instability in a viscoelastic flow

$$\left.\begin{aligned} \partial_t h &= v_y^{(int)}, \\ \partial_t q &= \frac{\mathscr{S}}{\rho_0}\partial_x^2 h + 2Agh + \frac{2\nu\eta c}{\bar{\tau}}\Sigma + 2\nu\partial_x^2 q, \\ \partial_t \Sigma &= 2\partial_x^2 q - \frac{2}{\bar{\tau}}\Sigma - \left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right)\int dy \phi \partial_x \sigma_{12}, \end{aligned}\right\} \tag{3.13}$$

where $c = 4\mu_1\mu_2/(\mu_1 + \mu_2)^2 \leqslant 1$.

## 4. Potential flow closure for the interface velocity

The set of equations (3.13) is not closed because of the presence of the interface velocity $v_y^{(int)}$ and of the integral term in the equation for $\Sigma$. In order to close the system we exploit again the potential flow approximation for which $v_y = \partial_y \psi$.

Taking into account the boundary condition for $y \to \infty$, the potential can be written (e.g. for $y \geqslant 0$) as

$$\psi(x, y, t) = \int_0^\infty \mathrm{e}^{-ky + \mathrm{i}kx} \hat{\psi}(k, t)\, \mathrm{d}k + c.c., \tag{4.1}$$

where '^' denotes the Fourier transform, and therefore

$$v_y(x, y, t) = -\int_0^\infty k\, \mathrm{e}^{-ky + \mathrm{i}kx} \hat{\psi}(k, t)\, \mathrm{d}k + c.c., \tag{4.2}$$

$$q(x, t) = -2\int_0^\infty \mathrm{e}^{\mathrm{i}kx} \hat{\psi}(k, t)\, \mathrm{d}k + c.c. \tag{4.3}$$

and taking a flat interface, $y = 0$, at the leading order

$$v^{(int)}(x, t) = -\int_0^\infty k\, \mathrm{e}^{\mathrm{i}kx} \hat{\psi}(k, t)\, \mathrm{d}k + c.c. \tag{4.4}$$

Assuming consistently that

$$\sigma_{12}(x, y, t) = \int_0^\infty \mathrm{e}^{-ky+\mathrm{i}kx}\hat{\sigma}_{12}(k, t)\,\mathrm{d}k + c.c. \tag{4.5}$$

in the limit of small amplitudes one has $\int \mathrm{d}y\phi\partial_x\sigma_{12} = 0$, and the set of (3.13) for the Fourier coefficients becomes

$$\left.\begin{aligned} \partial_t\hat{h} &= \frac{k}{2}\hat{q}, \\ \partial_t\hat{q} &= -\frac{\mathscr{S}}{\rho_0}k^2\hat{h} + 2Ag\hat{h} + \frac{2\nu c\eta}{\bar{\tau}}\hat{\Sigma} - 2\nu k^2\hat{q}, \\ \partial_t\hat{\Sigma} &= -2k^2q - \frac{2}{\bar{\tau}}\hat{\Sigma}. \end{aligned}\right\} \tag{4.6}$$

Restricting first to the case without polymers ($\eta = 0$), the growth rate $\alpha_N$ of the perturbation is obtained by looking for a solution of the form $\hat{h} \sim \mathrm{e}^{\alpha_N t}$ which gives

$$\alpha_N = -\nu k^2 + \sqrt{\omega^2 + (\nu k^2)^2}, \tag{4.7}$$

where it has been defined as

$$\omega = \sqrt{Agk - \frac{\mathscr{S}}{2\rho_0}k^3}. \tag{4.8}$$

The expression (4.8) is the well-known growth rate for a Newtonian fluid in the limit of zero viscosity (Chandrasekhar 1961), while (4.7) is a known upper bound to the growth rate for the case with finite viscosity (Menikoff *et al.* 1977).

Let us now consider the case with polymers, i.e. $\eta > 0$. The growth rate $\alpha$ is given by the solution of

$$(\alpha\bar{\tau})^3 + 2(\alpha\bar{\tau})^2(1 + \nu k^2\bar{\tau}) + \alpha\bar{\tau}[4\nu(1 + c\eta)k^2\bar{\tau} - \omega^2\bar{\tau}^2] - 2\omega^2\bar{\tau}^2 = 0. \tag{4.9}$$

The general solution is rather complicated and not very enlightening. In the limit of stiff polymers, $\bar{\tau} \to 0$, one gets

$$\alpha_0 \equiv \lim_{\bar{\tau}\to 0}\alpha = -\nu(1 + c\eta)k^2 + \sqrt{\omega^2 + [\nu(1 + c\eta)k^2]^2}. \tag{4.10}$$

Comparing with (4.7) one sees that in this limit polymers simply renormalize solvent viscosity. This result is in agreement with the phenomenological definition of $c\eta$ as the zero-shear polymer contribution to the total viscosity of the mixture (Virk 1975). Therefore, in order to quantify the effects of elasticity on RT instability, the growth rate for viscoelastic cases at finite $\bar{\tau}$ has to be compared with the Newtonian case with renormalized viscosity $\nu(1 + c\eta)$.

Another interesting limit is $\bar{\tau} \to \infty$. In this case from (4.9) one easily obtains that the growth rate coincides with that of the pure solvent (4.7), i.e. $\alpha_\infty = \alpha_N$. The physical interpretation is that in the limit $\bar{\tau} \to \infty$ and at finite time for which polymer elongation is finite, the last term in (2.1) vanishes and one recovers the Newtonian case without polymers (i.e. $\eta = 0$). Of course, this does not mean that in general polymer effects for high elasticity disappear. Indeed in the long-time limit polymer elongation is able to compensate the $1/\tau$ coefficient and in the late, nonlinear stages, one expects to observe strong polymer effects at high elasticity.

From (4.9) one can easily show (using implicit differentiation) that $\alpha(\bar{\tau})$ is a monotonic function and, because $\alpha_\infty \geqslant \alpha_0$, we have that instability rate grows with the elasticity, or the Deborah number, here defined as $De \equiv \omega\bar{\tau}$.

The case of stable stratification, $g \to -g$, is obtained by $\omega^2 \to -\omega^2$ neglecting surface tension. In this case (4.9) has no solution for positive $\alpha$, therefore polymers alone cannot induce instabilities in a stably stratified fluid.

## 5. Numerical results

The analytical results obtained in the previous sections are not exact as they are based on a closure obtained from the potential flow approximation. While this approximation is consistent for the inviscid limit $\nu = 0$ (where it gives the correct result (4.8) for a Newtonian fluid) for finite viscosity we have shown that it gives a known upper bound to the actual growth rate of the perturbation (Menikoff *et al.* 1977) (this is because the potential flow approximation underestimates the role of viscosity which reduces the instability). Nonetheless, in the case of Newtonian fluid this upper bound is known to be a good approximation of the actual value of the growth rate measured in numerical simulations (Menikoff *et al.* 1977). Because both $\bar{\tau} \to 0$ and $\bar{\tau} \to \infty$ limits correspond to Newtonian fluids, we expect that also in the viscoelastic case the potential flow description is a good approximation.

To investigate this important point, we have performed a set of numerical simulations of the full model (2.1)–(2.3) in the limit of constant viscosity and relaxation time (i.e. $\mu_1 = \mu_2$, $c = 1$ and $\tau_1 = \tau_2 = \bar{\tau}$) in two dimensions by means of a standard fully dealiazed pseudospectral method on a square doubly periodic domain. The resolution of the simulations is $1024 \times 1024$ collocation points (a comparative run at double resolution did not show substantial modifications on the results). More details on the numerical simulation method can be found in Celani, Mazzino & Vozella (2006) and Celani *et al.* (2009).

The basic state corresponds to a zero velocity field, a hyperbolic-tangent profile for the phase field and an uniform distribution of polymers in equilibrium, according to (2.7). The interface of the basic state is perturbed with a sinusoidal wave at wavenumber $k$ (corresponding to maximal instability for the linear analysis) of amplitude $h_0$ much smaller than the wavelength ($kh_0 = 0.05$).

The growth rate $\alpha$ of the perturbation is measured directly by fitting the height of the perturbed interface at different times with an exponential law. For given values of $A\,g$, $\mathscr{S}/\rho_0$, $\nu$ and $\eta$, this procedure is repeated for different values of $\bar{\tau}$ at the maximal instability wavenumber $k$ (which, for the range of parameters considered here, is always $k = 1$, i.e. it is not affected by elasticity). Figure 1 shows the results for two sets of runs at different values of $\eta$ and $\nu$. As discussed above, we find that the theoretical prediction given by (4.9) is indeed an upper bound for the actual growth rate of the perturbation. Nevertheless, the bound gives grow rates which are quite close to the numerical estimated values (the error is of the order of 10 %). The error is smaller for the runs having a larger value of $\eta$ and $\nu$, as was already discussed by Celani *et al.* (2009).

Both theoretical and numerical results show that the effect of polymers is to increase the perturbation growth rate. $\alpha$ grows with the elasticity and saturates for sufficiently large value of $De$.

## 6. Conclusions and perspectives

We investigated the role of polymers on the linear phase of the RT instability in an Oldroyd-B viscoelastic model. In the limit of vanishing Deborah number (i.e. vanishing polymer relaxation time) we recover a known upper bound for the growth

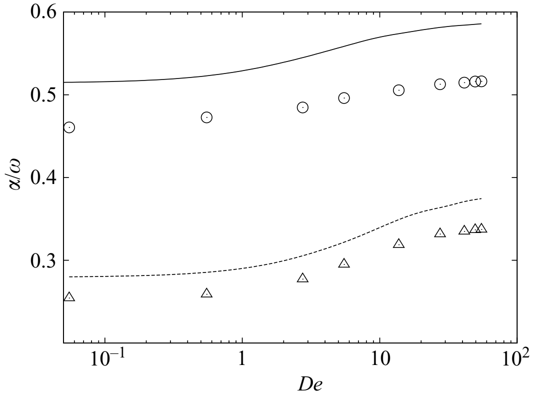

FIGURE 1. The perturbation growth rate $\alpha$ normalized with the inviscid growth rate $\omega$ (4.8) as a function of the Deborah number $De = \omega\bar{\tau}$. Points are the results of numerical simulations of the full set of equations (2.1)–(2.3), lines represent the theoretical predictions obtained from (4.9). The values of parameters are $c = 1$, $k = 1$, $Ag = 0.31$, $\mathscr{S}/\rho_0 = 0.019$ and $\eta = 0.3$, $\nu = 0.3$ (upper points and line) and $\eta = 0.5$, $\nu = 0.6$ (lower points and line).

rate of the perturbation in a viscous Newtonian fluid with modified viscosity. For finite elasticity, the growth rate is found to increase monotonically with the Deborah number reaching the solvent limit for high Deborah numbers. Our findings are corroborated by a set of direct numerical simulations on the viscoelastic Boussinesq Oldroyd-B model.

Our analysis has been confined to the linear phase of the perturbation evolution. When the perturbation amplitude becomes sufficiently large, nonlinear effects enter into play and a fully developed turbulent regime rapidly sets in (Cabot & Cook 2006; Boffetta *et al.* 2009; Vladimirova & Chertkov 2009). In the turbulent stage we expect more dramatic effects of polymers. In turbulent flows, a spectacular consequence of viscoelasticity induced by polymers is the drag reduction effect: addition of minute amounts (a few tenths of p.p.m. in weight) of long-chain soluble polymers to water leads to a strong reduction (up to 80 %) of the power necessary to maintain a given throughput in a channel (see e.g. Toms 1949; Virk 1975). We conjecture that a similar phenomenon might arise also in the present context. Heuristically, the RT system can indeed be assimilated to a channel inside which vertical motion of thermal plumes is maintained by the available potential energy. This analogy suggests the possibility to observe in the viscoelastic RT system a ‘drag’ reduction (or mixing enhancement) phenomenon, i.e. an increase of the velocity of thermal plumes with respect to the Newtonian case. Whether or not this picture does apply to the fully developed turbulence regime is left for future research.

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